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Fuzzy Simply Lindelöf Space and Fuzzy Simply Baire Space

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Abstract: In this paper we discussed about the relations of fuzzy Lindelöf space and fuzzy Baire Space, fuzzy simply α-Lindelöf space and fuzzy α-Baire space, fuzzy simply pre-Lindelöf space and fuzzy pre-Baire space, fuzzy simply semi-Lindelöf space and fuzzy semi-Baire space. Some characterizations of fuzzy simply Lindelöf space and fuzzy Baire spaces are also studied. Several examples are given to illustrate these relations.

Keywords: Fuzzy simply open set, fuzzy dense set, fuzzy nowhere dense set, fuzzy first category, fuzzy second category, fuzzy simply Lindelöf space, fuzzy Baire spaces.

1. INTRODUCTION:

The fuzzy concept has invaded almost all branches of Mathematics ever since the introduction of fuzzy set by L.A.ZADEH. The theory of fuzzy topological spaces was introduced and developed by C.L.CHANG. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. In this paper the relations of fuzzy simply Lindelöf space and fuzzy Baire space, fuzzy simply α-Lindelöf space and fuzzy α-Baire space, fuzzy simply pre-Lindelöf space and fuzzy pre-Baire space, fuzzy simply semi-Lindelöf space and fuzzy semi-Baire space are discussed and some characterizations of fuzzy simply Lindelöf space and fuzzy Baire spaces are studied. Several examples are given to illustrate these relations.

2. PRELIMINARIES:

In this section, we recall the basic definitions.

Fuzzy simply open set:[5]

If λ is a fuzzy open and fuzzy dense set in a fuzzy topological space (X,T), then λ is a fuzzy simply open set in (X,T).

Fuzzy dense set:[5]

A fuzzy set λ in a fuzzy topological space (X,T), is called fuzzy dense if there exist no non-zero fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. (i.e) $cl(\lambda) = 1$

Fuzzy nowhere dense set: [5]

A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exist no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$, i.e. $\operatorname{int} cl(\lambda) = 0$ in (X,T).

Fuzzy simply Lindelöf space:[5]

A fuzzy topological space (X,T) is said to be fuzzy simply Lindel**ö**f if each cover of X by fuzzy simply open sets has a countable sub cover. That is , (X,T) is a fuzzy simply Lindel**ö**f space if $\vee_{\alpha\in\Delta} \{\lambda_\alpha\} = 1$.

Fuzzy Baire space:[1]

Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy Baire space if, int($\vee_{i=1}^{\infty} \lambda_i$) = 0 where λ_i's are fuzzy nowhere dense sets in (X,T).

Fuzzy α-Baire space:[2]

Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy α- Baire space if,α- int($\vee_{i=1}^{\infty} \lambda_i$) = 0 where λ_i's are fuzzy α-nowhere dense sets in (X,T).

Fuzzy pre-Baire space:[3]

Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy pre- Baire space if, pint($\vee_{i=1}^{\infty} \lambda_i$) = 0 where λ_i's are fuzzy pre-nowhere dense sets in (X,T).

Fuzzy semi-Baire space:[4]

Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy semi- Baire space if, sint($\vee_{i=1}^{\infty} \lambda_i$) = 0 where λ_i's are fuzzy semi-nowhere dense sets in (X,T).

3. FUZZY SIMPLY LINDELÖF SAPCE AND FUZZY BAIRE SPACE:

A fuzzy topological space (X,T) is said to be fuzzy simply Lindelöf if each cover of X by fuzzy simply open sets has a countable sub cover. That is , (X,T) is a fuzzy simply Lindelöf space if $\vee_{\alpha\in\Delta}\{\lambda_\alpha\}=1$.

Example 3.1: Let X={a,b,c}. The fuzzy sets $\lambda, \mu, \delta, \alpha, \beta,$ and γ are defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.5; λ(b) = 0.6; λ(c) =1;
$\mu : X \to [0, 1]$ is defined as μ(a) = 0.7; μ(b) = 1 ; μ(c) = 0.7;
$\delta : X \to [0, 1]$ is defined as δ(a) = 1; δ(b) = 0.6; δ(c) = 0.7;
$\alpha : X \to [0, 1]$ is defined as α(a) = 0.6; α(b) = 0.8;α(c) = 1
$\beta : X \to [0,1]$ is defined as β(a) = 0.6;β(b) = 1;β(c) = 0.7;
$\gamma : X \to [0, 1]$ is defined as γ(a) = 1;γ(b) = 0.6;γ(c) = 0.

$$T = \{0, \lambda, \mu, \delta, \lambda\vee\mu, \lambda\vee\delta, \mu\vee\delta, \lambda\wedge\mu, \lambda\wedge\delta, \mu\wedge\delta, \lambda\vee(\mu\wedge\delta), \mu\wedge(\lambda\vee\delta), \delta\vee(\lambda\vee\mu), 1\}$$

is fuzzy topology on X. On computation, we see that the fuzzy set

$$\{\lambda, \mu, \delta, \lambda\vee\mu, \lambda\vee\delta, \mu\vee\delta, \lambda\wedge\mu, \lambda\wedge\delta, \mu\wedge\delta, \lambda\vee(\mu\wedge\delta), \mu\wedge(\lambda\vee\delta), \delta\vee(\lambda\vee\mu)\}$$

are fuzzy open and fuzzy dense sets in (X, T). Now

$$\lambda, \mu, \delta, \lambda\vee\mu, \lambda\vee\delta, \mu\vee\delta, \lambda\wedge\mu, \lambda\wedge\delta, \mu\wedge\delta, \lambda\vee(\mu\wedge\delta), \mu\wedge(\lambda\vee\delta), \delta\vee(\lambda\vee\mu)$$

are fuzzy simply open sets in (X,T). Also cl(α) =cl(β) = cl(γ) = cl(1-γ) = 1; cl(1- α) = 1-λ ; cl(1-β) = 1-($\lambda\wedge\delta$) ; int(α) = λ ; int(β) = $\mu\wedge\delta$;int(γ) = int(1-α) = int(1-β) = int(1-γ) = 0. Now intcl[cl(α) ∧ cl(1- α)] = 0, intcl[cl(β) ∧ cl(1-β)] = 0. intcl[cl(γ) ∧ cl(1-γ)] = 1 ≠ 0 and hence α,β are fuzzy simply open sets in (X,T).Now for the cover $\{\lambda, \lambda\vee\mu, \lambda\vee\delta, \mu\vee\delta, \delta\vee(\lambda\wedge\mu)\}$ of X by simply open sets $\{(\lambda\vee\mu)\vee(\lambda\vee\delta)\} = 1$ and hence for the cover {λ, β, γ, $\mu\vee\delta, \lambda\wedge\mu$} of X by simply open sets,{λ ∨ β ∨ γ} = 1 implies that (X,T) is a fuzzy simply Lindelöf space.

A fuzzy simply Lindelöf space is a fuzzy Baire space. Consider the following examples.

Example 3.2: Let X={a, b, c}. Consider the fuzzy sets λ ,μ and δ defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.5; λ(b) = 0.6; λ(c) =1;
$\mu : X \to [0, 1]$ is defined as μ(a) = 0.7; μ(b) = 1 ; μ(c) = 0.7;
$\delta : X \to [0, 1]$ is defined as δ(a) = 1; δ(b) = 0.6; δ(c) = 0.7;

$$T = \{0, \lambda, \mu, \delta, \lambda\vee\mu, \mu\vee\delta, \lambda\vee\delta, \lambda\wedge\mu, \lambda\wedge\delta, \mu\wedge\delta, \lambda\vee(\mu\wedge\delta), \mu\wedge(\lambda\vee\delta), \delta\vee(\lambda\vee\mu), 1\}$$

Now

$$1-\lambda, 1-\mu, 1-\delta, 1-\lambda\vee\mu, 1-\mu\vee\delta, 1-\lambda\vee\delta, 1-\lambda\wedge\mu, 1-\lambda\wedge\delta, 1-\mu\wedge\delta, 1-\lambda\vee(\mu\wedge\delta), 1-\mu\wedge(\lambda\vee\delta), 1-\delta\vee(\lambda\vee\mu)$$

are fuzzy nowhere dense sets in (X,T).
Now

$$[1-\lambda, 1-\mu, 1-\delta, 1-\lambda\vee\mu, 1-\mu\vee\delta, 1-\lambda\vee\delta, 1-\lambda\wedge\mu, 1-\lambda\wedge\delta, 1-\mu\wedge\delta, 1-\lambda\vee(\mu\wedge\delta), 1-\mu\wedge(\lambda\vee\delta), 1-\delta\vee(\lambda\vee\mu)] = 0.$$

and therefore, a fuzzy simply Lindelöf space is a fuzzy Baire space.

Preposition 3.4: If λ is a fuzzy simply Lindelöf space in (X,T). Such that, $\mu \le (1-\lambda)$ then μ is a fuzzy simply nowhere dense set in (X,T).

Proof: Let (X, T) be a fuzzy simply Lindelöf space. Then, $\vee_{\alpha\in\Delta}\{\lambda_\alpha\}=1$ .Where $\{\lambda_\alpha\}$'s fuzzy open and fuzzy dense set. Now $\mu \le (1-\lambda)$ such that, $cl\mu \le cl(1-\lambda)$. Then we have, $\operatorname{int} cl\mu \le \operatorname{int} cl(1-\lambda) \le \operatorname{int}(1-\lambda) = 1 - cl(\lambda) = 1-1 = 0$. Since 1-λ is closed and cl(λ)=1. Hence, μ is a fuzzy nowhere dense set in (X, T).

Theorem: 3.5. [1]. If λ is a fuzzy open and fuzzy dense set in a fuzzy topological space in (X,T). Then $(1-\lambda)$ is a fuzzy nowhere dense set in (X, T).

Preposition 3.6: If λ is a fuzzy simply Lindelöf space in (X,T). Then $(1-\lambda)$ is a fuzzy nowhere dense set in (X, T).

Proof: Follows by theorem 3.5.

Theorem: 3.7. [1]. If λ is a fuzzy nowhere dense set in a fuzzy topological space in (X,T). Then $(1-\lambda)$ is a fuzzy dense set in (X, T).

Preposition 3.8: If λ_i be a fuzzy nowhere dense set in simply Lindelöf space (X, T), then (1- λ_i) is fuzzy dense set in (X,T).

Proof: Follows by theorem 3.7.

4. FUZZY SIMPLY α -LINDELÖF SAPCE AND FUZZY α-BAIRE SPACE:

Fuzzy simply α -Lindelöf space.

A fuzzy topological space (X,T) is said to be fuzzy simply α- Lindel**ö**f space if each cover of X by fuzzy simply α -open sets has a countable sub cover. That is , (X,T) is a fuzzy simply α -Lindel**ö**f space if $\vee_{\alpha\in\Delta}\{\lambda_\alpha\}=1$

Example 4.1 : Let X={a,b,c}. The fuzzy sets $\lambda, \mu, \delta, \alpha, \beta,$ and γ is defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.5; λ(b) = 0.6; λ(c) =1;
$\mu : X \to [0, 1]$ is defined as μ(a) = 0.7; μ(b) = 1 ; μ(c) = 0.7;

$\delta : X \to [0, 1]$ is defined as δ(a) = 1; δ(b) = 0.6; δ(c) = 0.7;
$\eta : X \to [0, 1]$ is defined as η(a) = 0.6; η(b) = 0.8;η(c) = 1
$\beta : X \to [0,1]$ is defined as β(a) = 0.6;β(b) = 1;β(c) = 0.7;
$\gamma : X \to [0, 1]$ is defined as γ(a) = 1;γ(b) = 0.6;γ(c) = 0.

$T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta,$
$\lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu),1\}$
is fuzzy topology on X. On computation, we see that the fuzzy sets
$\{\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta),$
$\mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu)\ \}$
are fuzzy α-open and fuzzy α- dense sets in (X,T), then the fuzzy sets
$\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta),$
$\mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu)$
are fuzzy simply α- open sets in (X,T). Also cl(η) =cl(β) = cl(γ) = cl(1-γ) = 1; cl(1- η) = 1-λ ; cl(1-β) = 1-($\lambda \wedge \delta$) ; int(η) = λ ; int(β) = $\mu \wedge \delta$;int(γ) = int(1-η) = int(1-β) = int(1-γ) = 0. Now intcl[cl(η) $\wedge$ cl(1- η)] = 0, intcl[cl(β) $\wedge$ cl(1-β)] = 0. intcl[cl(γ) $\wedge$ cl(1-γ)] = 1 $\neq$ 0 and hence η,β are fuzzy simply α- open sets in (X,T).Now for the cover
$\{\lambda, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta\ , \delta \vee (\lambda \wedge \mu)\}$ of X by simply α-open sets $\{(\lambda \vee \mu) \vee (\lambda \vee \delta)\}$ = 1 and hence for the cover
{λ, β, γ, $\mu \vee \delta$, $\lambda \wedge \mu$} of X by simply α- open sets,{λ $\vee$ β $\vee$ γ} = 1 implies that (X,T) is a fuzzy simply α- Lindelöf space.

A fuzzy simply α-Lindel**ö**f space is a fuzzy α-Baire space. Consider the following examples.

Example 4.2: Let X={a, b, c}. Consider the fuzzy sets λ ,μ and δ defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.5; λ(b) = 0.6; λ(c) =1;
$\mu : X \to [0, 1]$ is defined as μ(a) = 0.7; μ(b) = 1 ; μ(c) = 0.7;
$\delta : X \to [0, 1]$ is defined as δ(a) = 1; δ(b) = 0.6; δ(c) = 0.7;

$T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \mu \vee \delta, \lambda \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta,$
$\lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu),1\}$
Now
$1-\lambda, 1-\mu, 1-\delta, 1-\lambda \vee \mu, 1-\mu \vee \delta, 1-\lambda \vee \delta, 1-\lambda \wedge \mu, 1-\lambda \wedge \delta,$
$1-\mu \wedge \delta, 1-\lambda \vee (\mu \wedge \delta), 1-\mu \wedge (\lambda \vee \delta), 1-\delta \vee (\lambda \vee \mu)$
are fuzzy α- nowhere dense sets in (X, T). Now int $[1-\lambda, 1-\mu, 1-\delta, 1-\lambda \vee \mu, 1-\mu \vee \delta, 1-\lambda \vee \delta, 1-\lambda \wedge \mu, 1-\lambda \wedge \delta,$
$1-\mu \wedge \delta, 1-\lambda \vee (\mu \wedge \delta), 1-\mu \wedge (\lambda \vee \delta), 1-\delta \vee (\lambda \vee \mu)] = 0.$
Therefore, a fuzzy simply α- Lindelöf space is a fuzzy α - Baire space.

Preposition 4.3: If λ is a fuzzy simply α- Lindelöf space in (X,T), such that , $\mu \le (1-\lambda)$ then μ is a fuzzy α- nowhere dense set in (X,T).

Proof: Let (X,T) be a fuzzy simply α- Lindelöf space. Then, $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$.Where, $\{\lambda_\alpha\}$'s are fuzzy α- open and fuzzy α-dense set Now $\mu \le (1-\lambda)$ such that, $\alpha cl \mu \le \alpha cl(1-\lambda)$. Then we have, $\alpha \operatorname{int} cl \mu \le \alpha \operatorname{int} cl(1-\lambda)$ $\le \alpha \operatorname{int}(1-\lambda) = 1 - \alpha cl(\lambda) = 1-1 = 0$. Since $(1-\lambda)$ is fuzzy α- closed in (X,T) and cl(λ)=1. Hence, μ is a fuzzy α- nowhere dense se in (X,T).

Theorem: 4.4. [2]. If λ is a fuzzy α-open and fuzzy α-dense set in a fuzzy topological space in (X,T). Then $(1-\lambda)$ is a fuzzy α-nowhere dense set in (X, T).

Preposition 4.5: If λ is a fuzzy α-simply Lindelöf space in (X,T). Then $(1-\lambda)$ is a fuzzy α-nowhere dense set in (X,T).

Proof : Follows from theorem 4.4.

Theorem 4.6: [1] If λ is fuzzy nowhere dense set in (X,T), then int(λ) = 0.

Preposition 4.7: If λ be a fuzzy α- nowhere dense set in Lindelöf space (X, T). Then (1- λ_i) is fuzzy α-dense set in (X, T).

Proof: Let λ be a fuzzy α-nowhere dense set in (X, T). Then by theorem [4.3] , "If λ is fuzzy nowhere dense set in (X,T), then int(λ) = 0." Let λ be a fuzzy α- nowhere dense set in (X,T). Now, $\lambda_i \le \alpha cl(\lambda_i)$ implies that $\alpha \operatorname{int}(\lambda_i) \le \alpha \operatorname{int} \alpha cl(\lambda_i) = 0$ then $\alpha \operatorname{int}(\lambda_i) = 0$. Now 1-α-int($\lambda_i$) = α-cl(1-λ) = 0. Hence 1-λ_i 's are fuzzy dense set in (X, T).

5. FUZZY SIMPLY PRE-LINDELÖF SAPCE AND FUZZY PRE-BAIRE SPACES:

Example 5.1 : Let X={a, b, c}. The fuzzy sets $\lambda, \mu, \delta, \alpha, \beta,$ and γ are defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.5; λ(b) = 0.6; λ(c) =1;
$\mu : X \to [0, 1]$ is defined as μ(a) = 0.7; μ(b) = 1 ; μ(c) = 0.7;
$\delta : X \to [0, 1]$ is defined as δ(a) = 1; δ(b) = 0.6; δ(c) = 0.7;
$\alpha : X \to [0, 1]$ is defined as α(a) = 0.6; α(b) = 0.8;α(c) = 1
$\beta : X \to [0,1]$ is defined as β(a) = 0.6;β(b) = 1;β(c) = 0.7;
$\gamma : X \to [0, 1]$ is defined as γ(a) = 1;γ(b) = 0.6;γ(c) = 0.

$T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta,$
$\lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu),1\}$
is fuzzy topology on X. On computation, we see that the fuzzy sets

$\{\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta),$
$\mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu)\ \}$
are fuzzy pre- open and fuzzy pre- dense sets in (X,T). Now the fuzzy sets
$\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta),$
$\mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu)$
are fuzzy simply pre-open sets in (X,T). Also pcl(α) =pcl(β) = pcl(γ) = pcl(1-γ) = 1; pcl(1- α) = 1-λ ; pcl(1-β) = 1-($\lambda \wedge \delta$) ; pint(α) = λ ; pint(β) = $\mu \wedge \delta$;pint(γ) = pint(1-α) = pint(1-β) = pint(1-γ) = 0. Now pintpcl[cl(α) $\wedge$ cl(1- α)] = 0, pintpcl[cl(β) $\wedge$ cl(1-β)] = 0. pintpcl[cl(γ) $\wedge$ cl(1-γ)] = 1 $\neq$ 0 and hence α,β are fuzzy simply pre-open sets in (X,T).Now for the cover $\{\lambda, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta\ , \delta \vee (\lambda \wedge \mu)\}$ of X by simply pre-open sets $\{(\lambda \vee \mu) \vee (\lambda \vee \delta)\} = 1$ and hence for the cover {λ, β, γ, $\mu \vee \delta, \lambda \wedge \mu$} of X by simply pre-open sets,{λ $\vee$ β $\vee$ γ} = 1 implies that (X,T) is a fuzzy simply pre-Lindelöf space.

A fuzzy simply pre-Lindelöf space is a fuzzy pre-Baire space. Consider the following examples.

Example 5.2: Let X = {a, b, c } and λ ,μ, δ be the fuzzy sets defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.5; λ(b) = 0.6; λ(c) =1;
$\mu : X \to [0, 1]$ is defined as μ(a) = 0.7; μ(b) = 1 ; μ(c) = 0.7;
$\delta : X \to [0, 1]$ is defined as δ(a) = 1; δ(b) = 0.6; δ(c) = 0.7;

$T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \mu \vee \delta, \lambda \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta,$
$\lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu), 1\}$
Now
$1-\lambda, 1-\mu, 1-\delta, 1-\lambda \vee \mu, 1-\mu \vee \delta, 1-\lambda \vee \delta, 1-\lambda \wedge \mu, 1-\lambda \wedge \delta,$
$1-\mu \wedge \delta, 1-\lambda \vee (\mu \wedge \delta), 1-\mu \wedge (\lambda \vee \delta), 1-\delta \vee (\lambda \vee \mu)$
are fuzzy pre- nowhere dense sets in (X,T).
Now
$[1-\lambda, 1-\mu, 1-\delta, 1-\lambda \vee \mu, 1-\mu \vee \delta, 1-\lambda \vee \delta, 1-\lambda \wedge \mu, 1-\lambda \wedge \delta,$
$1-\mu \wedge \delta, 1-\lambda \vee (\mu \wedge \delta), 1-\mu \wedge (\lambda \vee \delta), 1-\delta \vee (\lambda \vee \mu)] = 0.$
And therefore, a fuzzy simply pre- Lindelöf space is a fuzzy pre-Baire space.

Preposition 5.3: If λ is a fuzzy simply pre- Lindelöf space in (X,T) such that , $\mu \le (1-\lambda)$ then μ is a fuzzy pre- nowhere dense set in (X,T).

Proof: Let (X, T) be a fuzzy simply pre- Lindelöf space. Then, $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$. Where, $\{\lambda_\alpha\}$'s are fuzzy pre- open and fuzzy pre-dense set. Now $\mu \le (1-\lambda)$ such that, $pcl\mu \le pcl(1-\lambda)$.Then we have, $p\operatorname{int} pcl\mu \le p\operatorname{int} pcl(1-\lambda)$ $\le p\operatorname{int}(1-\lambda) = 1 - pcl(\lambda) = 1-1 = 0$. Since $(1-\lambda)$ is fuzzy pre- closed in (X,T). Hence, μ is a fuzzy pre- nowhere dense se in (X,T).

Theorem: 5.4.[3]. If λ is a fuzzy pre-open and fuzzy pre-dense set in a fuzzy topological space in (X,T). Then $(1-\lambda)$ is a fuzzy pre-nowhere dense set in (X, T).

Preposition 5.5. If λ is a fuzzy pre-simply Lindelöf space in (X,T). Then $(1-\lambda)$ is a fuzzy pre-nowhere dense set in (X,T).

Proof : Follwes from theorem 5.4.

Preposition 5.6: If λ_i be a fuzzy pre- nowhere dense set in Lindelöf space (X,T),then (1- λ) is fuzzy pre-dense set in (X,T).

Proof: Let λ be a fuzzy pre-nowhere dense set in (X,T). Then pint(λ) = 0 .Now, $\lambda_i \le pcl(\lambda_i)$ implies that $p\operatorname{int}(\lambda_i) \le p\operatorname{int}\ pcl(\lambda_i) = 0$ then $p\operatorname{int}(\lambda_i) = 0$. we have $p\operatorname{int}(\lambda_i) = 0$. Now, $pcl(1-\lambda_i) = 1 - p\operatorname{int}(\lambda_i) = 1 - 0 = 1$.
Hence, $pcl(1-\lambda_i)$ is a fuzzy pre- dense set in (X,T).

6. FUZZY SIMPLY SEMI-LINDELÖF SAPCE AND FUZZY SEMI-BAIRE SPACES:

Example 6.1: Let X={a,b,c}. The fuzzy sets $\lambda, \mu, \delta, \alpha, \beta,$ and γ are defined on X as follows:
$\lambda : X \to [0, 1]$ is defined as λ(a) = 0.5; λ(b) = 0.6; λ(c) =1;
$\mu : X \to [0, 1]$ is defined as μ(a) = 0.7; μ(b) = 1 ; μ(c) = 0.7;
$\delta : X \to [0, 1]$ is defined as δ(a) = 1; δ(b) = 0.6; δ(c) = 0.7;
$\alpha : X \to [0, 1]$ is defined as α(a) = 0.6; α(b) = 0.8;α(c) = 1
$\beta : X \to [0, 1]$ is defined as β(a) = 0.6;β(b) = 1;β(c) = 0.7;
$\gamma : X \to [0, 1]$ is defined as γ(a) = 1;γ(b) = 0.6;γ(c) = 0.

$T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta,$
$\lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu), 1\}$
is fuzzy topology on X. On computation, we see that the fuzzy sets
$\{\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta),$
$\mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu)\ \}$
are fuzzy semi- open and fuzzy semi- dense sets in (X,T). The fuzzy sets
$\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta),$
$\mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu)$
are fuzzy simply semi- open sets in (X,T). Also scl(α) =scl(β) = scl(γ) = scl(1-γ) = 1; scl(1- α) = 1-λ ; scl(1-β) = 1-($\lambda \wedge \delta$) ; sint(α) = λ ; sint(β) = $\mu \wedge \delta$;sint(γ) = sint(1-α) = sint(1-β) = sint(1-γ) = 0. Now sintscl[cl(α) $\wedge$ cl(1- α)] = 0, sintscl[cl(β) $\wedge$ cl(1-β)] = 0. sintscl[cl(γ) $\wedge$ cl(1-γ)] = 1 $\neq$ 0 and hence α,β

are fuzzy simply semi- open sets in (X,T).Now for the cover $\{\lambda, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta , \delta \vee (\lambda \wedge \mu)\}$ of X by simply semi-open sets $\{(\lambda \vee \mu) \vee (\lambda \vee \delta)\} = 1$ and hence for the cover $\{\lambda, \beta, \gamma, \mu \vee \delta, \lambda \wedge \mu \}$ of X by simply semi- open sets, $\{\lambda \vee \beta \vee \gamma\} = 1$ implies that (X,T) is a fuzzy simply semi- Lindelöf space.

A fuzzy simply semi-Lindelöf space is a fuzzy semi-Baire space. Consider the following example.

Example 6.2: Let X = {a, b, c } and λ ,μ, δ be the fuzzy sets defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.6$; $\lambda(c) =1$;

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.7$; $\mu(b) = 1$; $\mu(c) = 0.7$;

$\delta : X \to [0, 1]$ is defined as $\delta(a) = 1$; $\delta(b) = 0.6$; $\delta(c) = 0.7$;

$$T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \mu \vee \delta, \lambda \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), \mu \wedge (\lambda \vee \delta), \delta \vee (\lambda \vee \mu), 1\}$$

Now

$$1-\lambda, 1-\mu, 1-\delta, 1-\lambda \vee \mu, 1-\mu \vee \delta, 1-\lambda \vee \delta, 1-\lambda \wedge \mu, 1-\lambda \wedge \delta, 1-\mu \wedge \delta, 1-\lambda \vee (\mu \wedge \delta), 1-\mu \wedge (\lambda \vee \delta), 1-\delta \vee (\lambda \vee \mu)$$

are fuzzy semi- nowhere dense sets in (X,T).

Now

$$[1-\lambda, 1-\mu, 1-\delta, 1-\lambda \vee \mu, 1-\mu \vee \delta, 1-\lambda \vee \delta, 1-\lambda \wedge \mu, 1-\lambda \wedge \delta, 1-\mu \wedge \delta, 1-\lambda \vee (\mu \wedge \delta), 1-\mu \wedge (\lambda \vee \delta), 1-\delta \vee (\lambda \vee \mu)] = 0.$$

And therefore, a fuzzy simply semi- Lindelöf space is a fuzzy semi-Baire space.

Preposition 6.3: If λ is a fuzzy simply semi- Lindelöf space in (X,T) such that , $\mu \leq (1-\lambda)$ then μ is a fuzzy semi-nowhere dense set in (X,T).

Proof: Let (X, T) be a fuzzy simply semi- Lindelöf space. Then, $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$. Where, $\{\lambda_\alpha\}$'s are fuzzy semi- open and fuzzy semi-dense set. Now $\mu \leq (1-\lambda)$ such that, $scl\mu \leq scl(1-\lambda)$. Then we have, $s\,\mathrm{int}\, scl\mu \leq \sin tcl(1-\lambda) \leq \sin t(1-\lambda) = 1 - scl(\lambda) = 1-1 = 0$. Since $(1-\lambda)$ is fuzzy semi- closed in (X,T). Hence, μ is a fuzzy semi- nowhere dense set in (X,T).

Preposition 6.4: If λ is a fuzzy semi-simply Lindelöf space in (X,T). Then $(1-\lambda)$ is a fuzzy semi-nowhere dense set in (X,T).

Proof : Let (X,T) be a fuzzy simply semi-Lindelöf space. . Then, $\vee_{\alpha \in \Delta} \{\lambda_\alpha\} = 1$ Where, $\{\lambda_\alpha\}$'s are fuzzy semi- open and fuzzy semi-dense set. Now,

$$\sin tscl(1-\lambda) \leq \sin tscl(1) - \sin tscl(\lambda) \leq 1 - \sin tscl(\lambda)$$

$$= 1 - scl \sin t(\lambda) = 1 - scl(\lambda) = 1-1 = 0.$$

Hence, $(1-\lambda)$ is a fuzzy semi-nowhere dense set in (X,T)

Theorem: 6.5. [4]. If λ is a fuzzy semi-nowhere dense set in a fuzzy topological space in (X,T). Then $(1-\lambda)$ is a fuzzy semi-dense set in (X, T).

Preposition 6.6: If λ_i be a fuzzy semi- nowhere dense set in simply Lindelöf space (X,T), then $(1- \lambda_i)$ is fuzzy semi-dense set in (X,T).

Proof: Follows from theorem 6.5.

CONCLUSION:

We are discussed about the relations of fuzzy simply Lindelöf space and fuzzy Baire space, fuzzy simply α-Lindelöf space and fuzzy α-Baire space, fuzzy simply Pre-Lindelöf space and fuzzy Pre-Baire space, fuzzy simply semi-Lindelöf space and fuzzy semi-Baire spaces are studied several examples are given to illustrate these relations.

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